Modelling the Mining Equipment Market Trend by Field Theory

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ABSTRACT

Using the Beckman's field theory of spatial economics, the flows of main products are analyzed between demand and offer, in the mining equipment market, and the segmentation of the world market is put to evidence. The data regarding the market suppliers, customers, trade intensity and average flows from published reports are used for extracting useful conclusions.

Keywords: Variational calculus; mining; economical space; industry; marketing.

1. INTRODUCTION

The market of industrial products, subject for B2B marketing, is a dispersed one because the producers and the customers are territorial dispersed in space. The sale-purchase-acquisition-supply process is scrolled in time and space [1,2]. The models of establishing the balance between offer and demand ignores the spatiality and makes the assumption that the marketing process is an instant process and the aspects of shipping costs are separately evaluated, eventually.

The classical methods are based on „punctual“ economy and usually ignores the aspect of geographical location of agents and markets [3,4]. The essential problems of economical...
analyze what, how and for whom to be produced, are studied without calculations of the distances, the shipping costs and other shortcomings generated by the spatial dimensions of the market. These issues are removed by the spatial economics models [5].

2. METHODS OF WORK

There are two approaches in modeling of spatial economy: the discrete approach or the continuous one. The discrete approach dominates the specialized literature, because the practical applications operate with a finite set of data.

For this approach, space is considered as a finite set of nodes, where the economical activities are involved (production and/or consumption); the nodes are interconnected by arches or communication links. All the links attributes as incidence, shipping costs, flows, can be represented through matrices.

But this algebraic layout does not have a geometrical significance of spatiality and specific attributes such as: size, magnitude, shape.

Alternatively, the space can be considered as a continuous, bidimensional geographical space, map projected for economical activities that can be placed anywhere and the product flows are described as plane vector fields, having variable size and course in space. That was the stand of the classics of spatial economy. Beckmann (1952) took the first step to the path of bidimensional approach in his well known theory of continuous space of market, relative to one product or asset.

His model uses scale fields and vector fields. Writing the coordinates of Euclidian space \(x_1, x_2\), it is to consider the distribution of price in this space as \(\lambda(x_1, x_2)\). The mark \(\lambda\) introduced by Beckmann comes from Lagrange multiplier of minimised costs.

From geometrical point of view, this is a surface defined on the two dimensional space (plain) \(x_1, x_2\). From mathematical point of view this is a scalar field.

Beckmann’s model uses another two scalar fields, local excess of demand \(z(x_1, x_2)\), which is the excess of consumption in local manufacturing, considered here being dependent only to location, generally depending on price \(z(x_1, x_2, \lambda)\), and the local shipping costs \(k(x_1, x_2)\). Written this way, the shipping cost is isotropic, meaning regardless of direction. A more general model is \(k(x_1, x_2, \theta)\), where \(\theta\) is the transport direction.

As it is already mentioned, the irregularity of road networks tend to equalize the directional irregularities, so the isotropic becomes a reasonable simplification.

The variation of shipping cost between locations depends only of local properties as the density of the network (influencing the necessity of detours that depart of Euclidian line) or the intensity of traffic, related to potential of crossing (as a decisive trigger of traffic delay as a result of traffic congestion).

Beside the mentioned scalar fields, we introduce vector fields, also. The last ones are bidimensional vectors, whose length and direction varies depending on the considered point of origin. Such vector field can be obtained immediately from the scalar field of price, thanks to its gradient, as:

\[
\nabla \lambda = \begin{pmatrix} \frac{\partial \lambda}{\partial x_1} \\ \frac{\partial \lambda}{\partial x_2} \end{pmatrix}
\]

(1)

Where \(\nabla\) is the usually sign for gradient. The vector field \(\nabla \lambda\) is the gradient of price, its direction indicates the growing trend of the most abrupt gradient of the surface distribution of price from that specific place to other areas of studied region, and its quota (ultimate magnitude) is:

\[
|\nabla \lambda| = \sqrt{\left(\frac{\partial \lambda}{\partial x_1}\right)^2 + \left(\frac{\partial \lambda}{\partial x_2}\right)^2}
\]

(2)

Represents the increase of price per unit of Euclidian distance in that abrupt direction of growth.

Besides that field derived from gradient, there is the need for another general vector field to represent the flow of traded merchandised. Meaning:

\[
\Phi = \left(\Phi_1(x_1, x_2), \Phi_2(x_1, x_2)\right)
\]

which quota:
\begin{align*}
|\Phi| &= \sqrt{(\Phi_1)^2 + (\Phi_2)^2}
\end{align*}

represents the volume of merchandise flow that cross the position \((x_1, x_2)\). Dividing the flow vector at its quota, we obtain the unitary length vector (versor):
\[
\Phi = \left( \frac{\Phi_1}{\sqrt{(\Phi_1)^2 + (\Phi_2)^2}}, \frac{\Phi_2}{\sqrt{(\Phi_1)^2 + (\Phi_2)^2}} \right) = (\cos\theta, \sin\theta)
\]

Obviously, \(|\Phi|\), \(\theta\) are the polar coordinates for \(\phi_1, \phi_2\), where \(|\Phi|\) is the amount of merchandise and \(\theta\) is the flow direction for each location. Both varies to coordinates \((x_1, x_2)\) of designated point.

Changing the amount of flow along the pathway must be related to excess of local inquiry, \(z(x_1, x_2)\).

To do so it is needed for divergence as differential processor, sustained by the following heuristic argumentation: let us consider the flow which crosses a small square having the sides \(dx_1\) and \(dx_2\) in flat field. The flow is considered to have the components \(\phi_1\) directed to the horizontal coordinate \(x_1\) and \(\phi_2\) directed to the vertical coordinate \(x_2\).

Crossing that infinitesimal rectangle, the horizontal component \(\phi_1\) modifies itself as \((\partial \phi_1 / \partial x_1) \cdot dx_1 \cdot dx_2\), while the vertical component \(\phi_2\) modifies itself \((\partial \phi_2 / \partial x_2) \cdot dx_2 \cdot dx_1\). That is why we obtain the total change:
\[
\nabla \cdot \phi = \left( \frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2} \right) dx_1 dx_2
\]

Or, normalizing for area unit:
\[
\nabla \cdot \phi = \left( \frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2} \right)
\]

The processor \(\nabla \cdot \phi\) (it is to observe the sign for the scalar multiplier) for the divergence must not be misidentified to the processor \(\nabla\) for gradient, the first applied to a vector field and has as result a scalar field, and the second applied to a scalar field having as result a vector field.

3. RESULTS AND DISCUSSION

Due to our vision, the divergence has the meaning of changing of the flow amount due to a source (density) in case of offer excess- which adds another quantity of product to immergent flow or a mine shaft (excess of demand), when its negative and reducesthe emmergent flow.

This is assured by the divergence theorem of Gauss, which links the flow and the divergence to the density of sources from the interior of a domain. Let us explain the previous by a few examples.

Example 1:

Similar to electric field produced by a concentrated charge or gravitational field produced by a mass, in the economic field an excess of offer produces a scalar field. Fig. 1. represents the intensity of the field which gradient creates current lines (b), and in plane \((x, y)\) can be seen equipotent lines (c).

![Fig. 1. Example of field for an offer excess in the center of a rectangular area, a) the intensity of field; b) the gradient flow and c) equipotent lines](image-url)
This field has decreasing electrostatic potential \( |\Phi| \sim 1/r = 1/(x_1^2 + x_2^2)^{1/2} \) given by a concentrated load or gravitational potential, given by a concentrated mass. The field is radial, divergent, decreasing intensity to distance, we consider a flow as \( \Phi = (x_1, x_2) \) which has the quota \( |\Phi| = (x_1^2 + x_2^2)^{1/2} \) and cohesive vectors on the path of the axis \( \Phi/|\Phi| = \left( \frac{x_1}{x_1^2 + x_2^2} \right)^{1/2}, \left( \frac{x_2}{x_1^2 + x_2^2} \right)^{1/2} \), which means that the flow is radial divergent and its volume grows in line to the origin distance.

We have the divergence
\[
\nabla \cdot \phi = \left( \frac{\partial x_1}{\partial x_1} + \frac{\partial x_2}{\partial x_2} \right) = 1 + 1 = 2
\]
and means we have an excess of offer consistently distributed, which enters into the merchandise flow through any point. If we consider a bounded surface that surround the area, this is an exporting surface because the divergence is positive.

Fig. 2. The lines of the radial divergent field of the increasing flow field related to the distance from the center (axis origin)

Fig. 3. Lines of hyperbolic field with zero divergence of the field, having an increasing flow related to the distance from the center (axis origin)
Example 2:

Let us consider a given flow by \( \Phi = (x_1, -x_2) \).

Its quota is still \( |\Phi| = (x_1^2 + x_2^2)^{1/2} \) and its vectors

\[
\Phi / |\Phi| = \left( \frac{x_1}{x_1^2 + x_2^2}^{1/2}, \frac{-x_2}{x_1^2 + x_2^2}^{1/2} \right).
\]

This is a cross-sectional flow along the hyperbolic lines. The volume is ascending to distance from the origin, but the divergance is

\[
\nabla \cdot \phi = \left( \frac{\partial x_1}{\partial x_1} + \frac{\partial x_2}{\partial x_2} \right) = 1 - 1 = 0,
\]

and this means that the offer and demand balance themselves at any point of the plane.

If we consider a bounded region, the trade flow just crosses it, with nothing to add and nothing to remove at local level. The volume of the flow is not constant, but increases by the distance from the origin. However nothing here is contradictory, the flow is not radial anymore and is constant along each line of the hyperbolic flow lines.

Beckmann’s marketing model on continuous plane [6].

The excess of demand (offer minus manufacturing) \( q \) can be defined as a positional function to plane \( q = q(x_1, x_2) \). This can be considered a scalar field in two dimensional plane, having the spatial coordinates \( x_1 \) și \( x_2 \), where a closed area \( A \) is defined.

The equilibrium requirement of the market is that in the whole \( A \) area to have:

\[
\iint_A q(x_1, x_2) \, dx_1 \, dx_2 = 0 \tag{3}
\]

With except of the case of absence of local trade when \( q(x_1, x_2) = 0 \) to any point, the condition (3) below urges the existence of some positive points of excessive demand, or negative ones (excess on offer). The movement of the merchandise is from the point of positive point of offer excess to the points of demand excess. The movement of the merchandise is described as a flow of a field.

To any point the field has a direction and a size, together define the flow of the vector \( \Phi = (x_1, x_2) \).

To the point where there is no manufacture or consumption or the local demand and offer are balanced, the flow does not exist. There more simultaneous directions of the flow to the particular points of the edge of domain, where there are manufacture centers or consumption centers. Those particular points are omitted from the integrals which determines inflows or outflows.

The relationship between flows and local values of the excess of demand are analog to the relationship between the flow of a liquid and sources, i.e sinkholes studied in hydrodynamics or the thermal flow in relationship to hot or cold sources studied in thermodynamics.

This relationship takes form of the well-known Laplace equation:

\[
\nabla \cdot \Phi = \left( \frac{\partial x_1}{\partial x_1} + \frac{\partial x_2}{\partial x_2} \right) = -q(x_1, x_2)
\]

In which \( \Phi(\Phi_1, \Phi_2) \) is the vectorial flow.

It is presumed that the normal flow which crosses the domain frontier is null.

If \( \lambda(x) \) is the price of the product in the location \( x \) and \( k(x) \) is the specific cost of transport (the cost of transporting the unit of production/the unit of distance form location \( x \)), it results in:

\[
|\text{grade} \lambda| = k \quad \text{or} \quad k\Phi / |\Phi| = \text{grade} \lambda.
\]

This is know as Beckmann’s equation [7].

The solution of the equation \( \Phi / |\Phi| = \text{grade} \lambda \) minimises the cost:

\[
\iint_A k |\phi| \, dx \, dy \tag{4}
\]

While fulfilling:

\[
\text{div} \, \phi + q(x, y) = 0.
\]

If we consider a function of utility \( u \) in synergy with what is written above we can produce:

\[
\Delta u = \text{div} \,(\text{grade} \, u) = -q \nabla \lambda / k
\]

The symbol \( \Delta \) is called Laplacian and has the expression:
\[ \Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} \]  

(5)

And the anterior equation is a variance of Poisson’s equation [8].

Poisson’s equation (5) results from maximizing the extremities of the functional (4), which is a cost function while fulfilling the restrictions of balance and is the application of a variational principle widely used in various domains.

If we know the regional distribution of prices \( \lambda(x_1, x_2) \), of transportation costs \( k(x_1, x_2) \) and the excess of supply (demand minus offer), \( q(x_1, y_2) \), which is considered a density of quantity, so it is punctually distributed through sources, producers i.e. sinkholes – consumers, the equation above is the classic equation of Poisson, in two dimensions, similar to the equation of heat transferrence on a conductive sheet with heat sources, i.e. concentrated heat losses, the right side having the significance of density (source power) while the left side (under the sign of the Laplacian, \( \Delta \)) is the temperature.

In our case, while maintaining know and canonized notations, \( u \) is a function of utility, whose gradient is a flow of products \( \Phi \), a vectorial filed with the components \( \Phi_x \) and \( \Phi_y \), or the size \( |\Phi| \) and the direction \( \theta \).

\[ \Phi = \nabla u \]

After solving Laplace’s equation, which is this form combines the equation of minimum of the shipping cost \( \int K(\phi)dxdy \) through \( k \phi / |\phi| = \text{grad} \lambda \) and the condition of balance of production-consumption-flow \( \text{div} \phi + q(x, y) = 0 \) we obtain the utility function \( u \) whose gradient gives us the flow (vectorial field).

Integrating the surface of the flow gives us the volume of products which crosses the frontier of the area. In our model, we replaced the intensity (the density) for every region, excess of the demand for every dipol, formed from supply and demand, separated, distanced with a random value, which represents the similar aspects of an electrical dipol.

It resides in a negative and positive load, i.e. a demand and a supply (the excess of demand being useless in perspective the inter-regional flow), especially is the areas in which the local offer partially satisfies the local demand in a random allocated quota, the remaining local demands being fulfilled by external sources, i.e. the excess of offer being redirected to external consumers.

It is given that this augmentation of the model boosts accuracy, but a further analysis should be made for the more exotic products, with a different mixture of supply and demand. While this analysis is fairly complicated, it is not of the impossible.

Fig. 4. Geographical areas that define the global market for minning machines
Table 1. The notations semnificance is given in the table below

<table>
<thead>
<tr>
<th>Region</th>
<th>Demand</th>
<th>Offer</th>
<th>Demand excess</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 NV North America(SUA+CANADA)</td>
<td>0.12</td>
<td>0.40</td>
<td>-0.28</td>
</tr>
<tr>
<td>2 SV Central and South America</td>
<td>0.10</td>
<td>0.11</td>
<td>-0.01</td>
</tr>
<tr>
<td>3 NC Europe (UE)</td>
<td>0.12</td>
<td>0.30</td>
<td>-0.18</td>
</tr>
<tr>
<td>4 SC Africa</td>
<td>0.06</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>5 NE China+Russia</td>
<td>0.20</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>6 SE Australia, Asia Pacific +India</td>
<td>0.40</td>
<td>0.03</td>
<td>0.37</td>
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<tr>
<td>Total</td>
<td>1.00</td>
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For the essay in question we’ve limited ourselves to analyzing the equipment needed for the extracting industry who are, by destination and functionality, similar, not taking into account other brands sold at similar prices, resulted from a natural balancing(long lasting), while the variety of production and consume trends of the areas in question being different.

The 6 geographical areas which caracterises the global market for mining machines are represented in Fig. 4.

In the Table 2, I’ve presented the global flows of products, extractive mining equipment, between supplyers and buyers from the 6 regions.

Using data from the charts, we manage to observe the distribution of sources and sinkholes like in the representation(I’ve used MathCad for generating the charts). What we discover is that every region is imbalanced from the point of supply and demand, while the global market remains stable.

Excess of offer has a global tendency to fall from West to East, while the supply trend rises from NorthWest to South East, observation reinforced by market studies made to this day.

By pursuing the study following the proposed method, we observe and are faced with the demonstration of the afromentioned trend, while justifying the opportunity of the method for market analysis of highly specific trend, from highly evolved regions to emerging areas where western technology maintains its popularity.

As consumers are, in the case of the mining industry, strictly bound to regions with rich mineral deposits, and the producers of mining equipment strongfully bound to areas with a tradition for mining, this fact becomes clear.

The cost of shipping is presented in the Fig. 6.

We have determined the prices gradient, necessary for coefficient calculations $\text{div}(\lambda)/k$ from Poisson’s equation.

Table 2. Global flows of products, extractive mining equipment, between supplyers and buyers from the 6 regions

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Through the averaging process, it is obtained the following, matrix of of density corection $\text{div}(\lambda)/k$ in the form of:

$$\text{div}(\lambda)/k = \begin{pmatrix} \text{coefNE} & \text{coefNC} & \text{coefNV} \\ \text{coefSE} & \text{coefSC} & \text{coefSV} \end{pmatrix} = \begin{pmatrix} 0.808 & 1.120 & 1.582 \\ 0.895 & 0.995 & 0.527 \end{pmatrix}$$
By solving Laplace’s equation, using MathCad procedure, we obtain the utility function $u$.

The peaks are the areas of offer excess, the lows are areas of excess demands.

As an intuitive interpretation, the offers/demand coincides with the altitude in a given tridimensional space under the action of a gravitational field, and the movement of products as trajectory of a particle with unitary mass, under the effect of said force field. The particle will move from peaks to bottoms, on the path with the least energy consumption, to the most attractive low following the field lines, with a speed proportional to it. In normalized values $u/\max(u)$ the diagram is as follows:

In Fig. 8 we observe the representation of iso-utility values in normalized values, similar to the equipotential lines of a field.

These curves can be interpreted as limits of influence of a certain producer, i.e. the economical limits of its influence on the market. Their geometry reflects the zoning proposed in the hypothesis, and their field density indicates the intensity of the flow of products.
Fig. 7. The solution to the Laplace equation which indicates the dispersion of offer and demand.

Fig. 8. Map of the iso-utility values in normalized values.

In Fig. 9 we are presented with the lines of current (direction of flow), which confirms the anterior presumption and can indicate the general direction of the flow to the regions (as example we have the NV area which is mainly supplying, while the SE area is mainly consuming).

These flows are in good measure with the bad tendencies.
Fig. 9. Current lines (Flow direction)

Fig. 10. The borders between supplying regions and consumer regions
The volume of products that are passed around can be determined from the partial flows that derive (or are sustained) from the center of the regions.

By determining the flow divergence, we find the borders between buying areas (positive divergence) and the supplying areas (negative divergence), separated by the lines div=0 which represents areas in which transportation is expensive or the price is low and doesn’t cover the shipping price, i.e. the areas with a balance between local supply and demand (Fig. 10).

Overlapping the two diagrams of the flow and divergence as in Fig. 11, the shown above are more eloquently confirmed.

Volume entered by trade is 9.85 valoric units (UV) while the exported volume in interregional trade is 5.42 UV and the difference 4.43 is the volume used internally in all regions.

The flow divergence is 0.9 (on all the market space) which represents a difference of aprox. 12% comparing to the value 0, a reasonable difference considering the precision level of the calculus.

We find the delimitation into 6 areas as follows: NV, NC, NE and SV, SC, SE

The analysis is useful, relevant and opportune from several points of view.

From a methodical point of view, it is a first step in the direction of applying the methods of spatial economics in the study of mining equipment market which are the point of interest for B2B marketing, products which occupy an important summ in the economics trade field (aprox. 100 billion dollars US in 2015 with a growth rate expectancy of 8-10% per year).

From an operational standpoint, the method can be used in market predictions, considering the dynamics of structural change in suppliers and consumers, dynamic which is expected to become more aggressive in the upcoming years.
4. CONCLUSION

From a knowledge point of view, it can be an bonding element between what we call operational marketing and scientologic aproach, taking the shape of a step forward in the field of applying spacial theory, theory that remained trapped in the field of regional economic evolution and the market for natural resources to the specific fields of our day, with a revigorating dynamic.

Marked as a begining, it can lead to an expert system of prediction/analyzing in the field of B2B marketing(using more extensve databases and computing power), filed that remains ancored in the conventional consumption marketing strategy of today.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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